g **i** local minimum at $(-\sqrt{2}, -1)$ and $(\sqrt{2}, -1)$, local maximum at $(0, 3)$,

ii non-horizontal inflection at $\left(\sqrt{\frac{2}{3}}, \frac{7}{9}\right)$

non-horizontal inflection at $\left(-\sqrt{\frac{2}{3}}, \frac{7}{9}\right)$

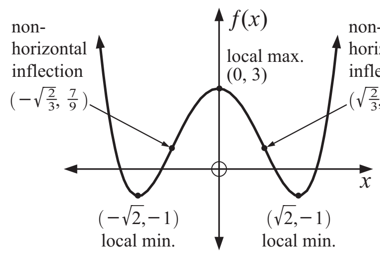
iii increasing for $-\sqrt{2} \leqslant x \leqslant 0$, $x \geqslant \sqrt{2}$

decreasing for $x \leqslant -\sqrt{2}$, $0 \leqslant x \leqslant \sqrt{2}$

iv concave down for $-\sqrt{\frac{2}{3}} \leqslant x \leqslant \sqrt{\frac{2}{3}}$

concave up for $x \leqslant -\sqrt{\frac{2}{3}}$, $x \geqslant \sqrt{\frac{2}{3}}$

v



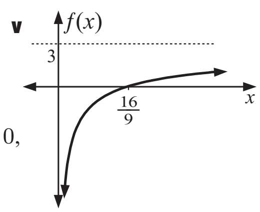
h **i** no stationary points

ii no inflections

iii increasing for $x > 0$, never decreasing

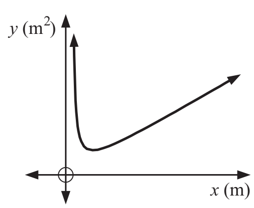
iv concave down for $x > 0$, never concave up

v



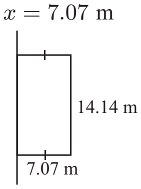
EXERCISE 22H

1 **b**



c $L_{min} = 28.28$ m, $x = 7.07$ m

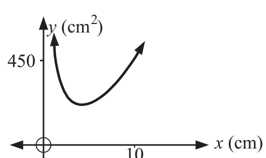
d



2 **a** $2x$ cm **b** $V = 200 = 2x \times x \times h$

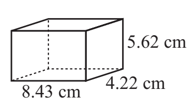
c **Hint**: Show $h = \dfrac{100}{x^2}$ and substitute into the surface area equation.

d



e $SA_{min} = 213.4$ cm², $x = 4.22$ cm

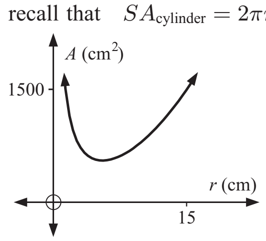
f



3 **a** recall that $V_{cylinder} = \pi r^2 h$ and that $1 \text{ L} = 1000 \text{ cm}^3$

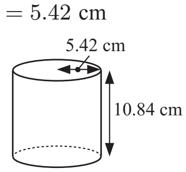
b recall that $SA_{cylinder} = 2\pi r^2 + 2\pi rh$

c



d $A = 554$ cm², $r = 5.42$ cm

e



4 **b** 6 cm × 6 cm

5 **a** $0 \leqslant x \leqslant 63.66$

c $x = 63.66$ m, $l = 0$ m (i.e., circular track)

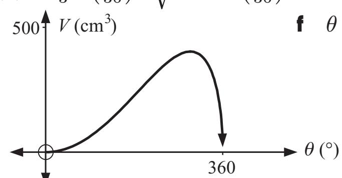
6 **a** **Hint**: Show that $\text{AC} = \frac{\theta}{360} \times 2\pi \times 10$

b **Hint**: Show that $2\pi r = \text{AC}$

c **Hint**: Use the result from **b** and Pythagoras' theorem.

d $V(\theta) = \frac{1}{3}\pi\left(\frac{\theta}{36}\right)^2 \sqrt{100 - \left(\frac{\theta}{36}\right)^2}$

e



f $\theta = 293.9^o$

7 **b** **Hint**: Show $C = 25x^2 + 200xy$ then use the result from **a**. **c** 1.59 m × 1.59 m × 0.397 m

8 **a** $2x$ units × $\dfrac{100}{x^2}$ units **b** **Hint**: Show that $\dfrac{dA}{dx} = -\dfrac{200}{x^2}$

c $P_{min} = 27.8$ units, 9.28 units × 4.64 units

9 13.44 cm from left (i.e., uses 13.44 cm for square tubing)

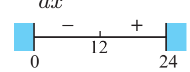
10 **a** For $x < 0$ or $x > 6$, X is not on AC.

c $x = 2.67$ km This is the distance from A to X which minimises the time taken to get from B to C. (Proof: Use sign diagram or second derivative test. Be sure to check the end points.)

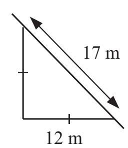
11 3.33 km **12** radius = 31.7 cm, height = 31.7 cm (Note: $100 \text{ L} = 0.1 \text{ m}^3$)

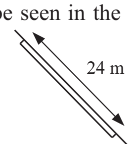
13 4 m from the 40 cp globe

14 **a** $D(x) = \sqrt{x^2 + (24-x)^2}$ **b** $\dfrac{d[D(x)]^2}{dx} = 4x - 48$



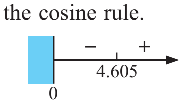
c Smallest $D(x) = 17.0$ Largest $D(x) = 24$, which is not an acceptable solution as can be seen in the diagram.





15 **a** **Hint**: Use the cosine rule.

b 3553 km²



c 5:36 pm

16 **a** $\text{QR} = \left(\dfrac{2+x}{x}\right)$ m **c** **Hint**: All solutions < 0 can be discarded as $x \geqslant 0$.

d 416 cm

17 between A and N, 2.566 m from N

18 at grid reference (3.544, 8) **19** $\text{A} = (4a, 0)$

20 $\sqrt{\frac{3}{2}} : 1$ **21** **e** 63.7%

EXERCISE 22I

1 **a** cost = \$878 000, average cost = \$1097.50, marginal cost = \$1850

b 195 items, \$639.87

2 **a** $A(x) = \frac{295}{x} + 24 - 0.08x + 0.0008x^2$

$C'(x) = 24 - 0.16x + 0.0024x^2$

b min. average cost = \$26.41 (when 79 items are made)

c min. marginal cost = \$21.33 (when 33 items are made)

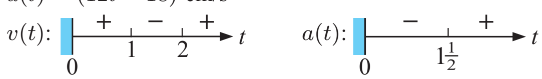
3 50 fittings **4** 250 items **5** 10 blankets

6 **a** $p(x) = 250 - \frac{x}{8}$, $x \geqslant 800$ **b** \$25 **c** \$10

7 25 km/h

REVIEW SET 22A

1 **a** $v(t) = (6t^2 - 18t + 12)$ cm/s
$a(t) = (12t - 18)$ cm/s^2

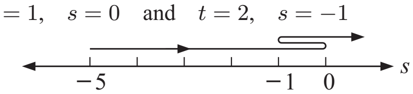


b $s(0) = 5$ cm to left of origin
$v(0) = 12$ cm/s towards origin
$a(0) = -18$ cm/s^2 (reducing speed)

c At $t = 2$, particle is 1 cm to the left of the origin, is stationary and is accelerating towards the origin.

d $t = 1$, $s = 0$ and $t = 2$, $s = -1$

e



f Speed is increasing for $1 \leqslant t \leqslant 1\frac{1}{2}$ and $t \geqslant 2$.

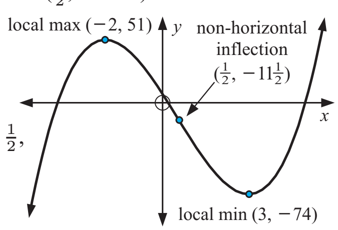
2 **a** **i** \$312 **ii** \$1218.75
b **i** \$9.10 per km/h **ii** \$7.50 per km/h **c** 3 km/h

3 **a** local maximum at $(-2, 51)$, local minimum at $(3, -74)$
non-horizontal inflection at $(\frac{1}{2}, -11.5)$

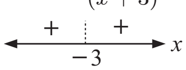
b increasing for $x \leqslant -2$, $x \geqslant 3$
decreasing for $-2 \leqslant x \leqslant 3$

c concave down for $x \leqslant \frac{1}{2}$,
concave up for $x \geqslant \frac{1}{2}$

d

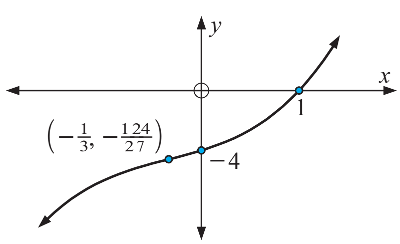


4 **a** $x = -3$
b y-intercept at $y = -\frac{2}{3}$, x-intercept at $x = \frac{2}{3}$
c $f'(x) = \frac{11}{(x+3)^2}$ **d** There are no stationary points.



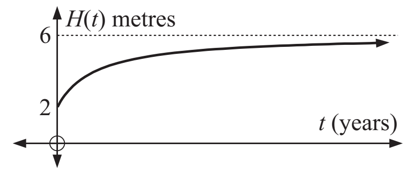
5 **b** $k = 9$

6 **a** $y = -4$ **b** $x = 1$ (only 1 intercept)
c no stat. points, non-horizontal inflection at $(-\frac{1}{3}, -\frac{124}{27})$
d



7 **a** $y = \frac{500}{x}$, $x > 0$ **b** $\frac{dy}{dx} = -\frac{500}{x^2}$ as $x^2 > 0$, $-\frac{500}{x^2} < 0$
c As the width of the rectangle increases, the length decreases.

8 **a** 2 m
b $H(3) = 4$ m, $H(6) = 4.67$ m, $H(9) = 5$ m
c $H'(0) = 1.33$ m/year, $H'(3) = 0.333$ m/year,
$H'(6) = 0.148$ m/year, $H'(9) = 0.083$ m/year
d As $H'(t) > 0$, the tree is always growing.
e



9 **a** $y = \frac{1}{x^2}$, $x > 0$
c base is 1.26 cm square, height is 0.630 m

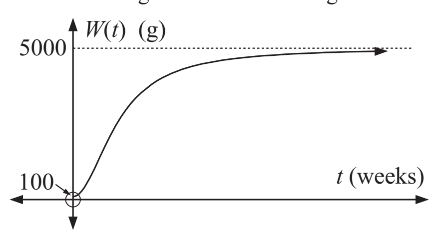
10 **b** $\frac{d[A(x)]^2}{dx} = 5000x - 4x^3$
Area is a maximum when $x \doteqdot 35.4$, $A = 1250$ m^2.

11 **a** $v(t) = 15 + \frac{120}{(t-1)^3}$ cm/s, $a(t) = \frac{-360}{(t-1)^4}$ cm/s^2
b At $t = 3$, particle is 30 cm to the right of the origin, moving to the right at 30 cm/s and decelerating at 22.5 cm/s^2.
c $0 \leqslant t < 1$

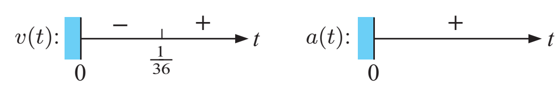
12 6 cm from each end

REVIEW SET 22B

1 **a** 100 g **b** **i** 2550 g **ii** 4711.8 g **iii** 4992.8 g
c **i** 0 **ii** 1514 g/week **iii** 333.5 g/week
d



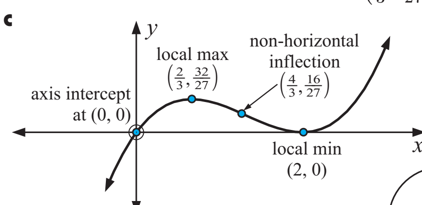
2 **a** $v(t) = 3 - \frac{1}{2\sqrt{t}}$, $a(t) = \frac{1}{4t\sqrt{t}}$



b $x(0) = 0$, $v(0)$ is undefined, $a(0)$ is undefined
c Particle is 24 cm to the right of the origin and is travelling to the right at 2.83 cm/s. Its speed is increasing.
d Changes direction at $t = \frac{1}{36}$, 0.083 cm to the left of the origin.
e Particle's speed is decreasing for $0 \leqslant t \leqslant \frac{1}{36}$.

3 **a** $t \geqslant 2$ **b** $t > 17$

4 **a** y-intercept at $y = 0$, x-intercept at $x = 0$ and $x = 2$
b local maximum at $\left(\frac{2}{3}, \frac{32}{27}\right)$, local minimum at $(2, 0)$, non-horizontal inflection at $\left(\frac{4}{3}, \frac{16}{27}\right)$
c



5 **b** $A(x) = 200x - 2x^2 - \frac{1}{2}\pi x^2$ **c**